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MATHEMATICAL MODEL OF TECHNOLOGICAL OPERATIONS ATTENDING DRIVAGE OF THE DEEP CURVILINEAR BORE-HOLES

МАТЕМАТИЧНА МОДЕЛЬ ТЕХНОЛОГІЧНИХ ОПЕРАЦІЙ БУРІННЯ ГЛИБОКИХ КРИВОЛІНІЙНИХ СВЕРДЛОВИН

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Summary: This article examines the process of deformation of the drill string, during which it bends and comes into contact with the walls of an oil and gas well, continuing to change its shape under the influence of increasing axial force, distributed contact load, and torque.

The purpose of the work is to construct nonlinear differential equations describing the contact interaction of the drill string pipe with the well wall, to propose a method for their numerical solution, and to present the results of computer modeling.

Research methods are theoretical and experimental.

This study develops a mathematical model to describe the mechanical interactions of a drill string with the wellbore during drilling, lowering, and raising operations. The model is based on the theory of curvilinear flexible rods and accounts for internal and external force factors, including contact and friction forces, gravity, and drilling fluid effects. A system of nonlinear differential equations is formulated to describe the stress-strain state of the drill string, incorporating geometric imperfections in the borehole trajectory. Computational simulations assess the impact of various borehole curvature conditions on drill string behavior, revealing significant influences on bending moments, frictional forces, and potential drill string seizure zones.

The practical application of the obtained results lies in improving methods for predicting emergency situations related to drill string sticking and developing recommendations for selecting optimal drilling parameters.

The obtained results make it possible to assess the likelihood of emergency situations, optimize drilling modes to enhance process efficiency, minimize equipment wear, and reduce the risks of accidents during the development of oil and gas fields.

Keywords: curvilinear drilling, borehole, geometrical imperfections, internal and external forces, direct and inverse problems.

Statement of the problem in general terms and its connection with important scientific and practical tasks. At present, approximately 90% of all energy consumed comes from fossil hydrocarbon fuels, with oil and gas being the primary contributors. Their prices are sharply rising due to increasingly complex extraction conditions and the approaching depletion of reserves. Nevertheless, the search for new oil and gas reserves and the progressively increasing rate of their extraction continue, underscoring the critical role of these resources in the global energy and economy. The principal technological component of these processes is the drilling of new oil and gas bores.

The most promising reserves of the remaining proven hydrocarbon resources are predominantly concentrated at great depths and in shelf zones far from dry land. This situation is compounded by the fact that, as a rule, only 40% of the hydrocarbon fuels that fill pores and cracks of underground deposits can be extracted using traditional technology, which involves drilling vertical boreholes. This low extraction efficiency leads to significant losses of valuable energy resources and creates both economic and environmental challenges.

An effective means to significantly enlarge extraction efficiency is to drill boreholes in inclined and horizontal manners. Since the early 2020s, advancements in drilling and completion technologies have made drilling these types of boreholes much more economical. Curvilinear boreholes allow penetration into oil- and gas-bearing strata along the laminated structure of the underground formations and covering larger zones of fuel output. This not only increases extraction volumes from existing fields but also opens access to previously inaccessible or economically unviable reserves.

Nevertheless, despite significant progress in the practical application of curvilinear drilling, the theoretical simulation methods for the mechanical effects accompanying the processes of curvilinear borehole drilling are not sufficiently advanced. There is a critical need for the development of more accurate and comprehensive theoretical models that account for the complex interactions between the drilling tool, rock, and fluid in non-linear trajectories.

Therefore, the development of advanced theoretical modeling methods for mechanical effects during curvilinear drilling is not only a scientific endeavor but also a critically important practical necessity for ensuring sustainable and economically efficient hydrocarbon extraction in the future. This problem is directly linked to energy security, economic stability, and reducing humanity's environmental footprint.

Analysis of publications in which research on this problem has been initiated and on which the authors rely. In work [1] the authors proposed a coil optimization method for ultra-deep electromagnetic logging while drilling, enhancing measurement accuracy in high-depth environments. The importance of precision directional drilling as a means of minimizing trajectory deviations and improving process efficiency was emphasized in [2].

A study explored the application of long-reach directional boreholes for gas drainage under complex geological conditions [3]. Experimental investigations were also conducted into the hydraulic fracturing properties of elliptical boreholes, simulating the effect of geometric anomalies [4].

Drill string buckling in complex boreholes was analysed, accounting for curvature-related effects [5]. A stochastic model of the nonlinear dynamic behaviour of horizontal drill strings, incorporating frictional and impact effects, was proposed [6]. Study [7] explores the key technologies and practical applications of deep borehole pneumatic directional drilling specifically adapted for challenging broken-soft coal seams. The paper [8] provides an analytical evaluation of the inertial properties of drill strings during their rotation.

The practical implementation of the technology for drilling wells of complex spatial orientation requires appropriate mathematical modelling for the design of their rational trajectories and the use of state-of-the-art equipment and technology for their drivage [9]. As this takes place, the most interesting issues are those for determining the external and internal forces as well as the torques acting on the drill strings in the curvilinear borehole in the processes of its tripping in/out and drilling [10]. Modelling resistance forces and dynamic phenomena accompanying well drilling makes it possible to solve such fundamental problems as obtaining the required trajectory of the well and reducing the longitudinal and transverse vibrations of the strings as well as reducing the forces of contact and friction interaction between the string and the well wall, thereby achieving a reduction in wear of the drill string and the lock joints, eliminating unplanned bending of the centreline of the well, and, as a consequence, eliminating complex abnormal situations in the drilling process[11]. These forces can only be calculated according to the theory of flexible curvature rods [12].

Presentation of the main research material. The borehole drivage process involves three main technological operations, differing by the operating regimes and by schemes of contact and friction interactions between the drill string (DS) and bore-hole surfaces. Among these are the drilling operation, which is main one, as well as the operations of the DS raising and lowering, performed for the change of dulled bits or for other technological needs. In performing each of them, the initiation of unforeseen and impermissible situations is possible, which are distinct from each other by combinations of force actions related to dissimilar manifestations and consequences.

With the aim to set up the problem about calculation of the external and internal quasi-static forces acting on a curvilinear DS at different stages of its functioning, take into account that usually the curvature radii of the bore-hole axes exceed hundreds of meters, the DS lengths equal several kilometers, but the clearances between the DS and bore-hole surfaces generally are 0,1 – 0,15 m. This permits one to assume that in the state of the DS operation its elastic line acquires the shape constrained by the bore-hole axis line and is prescribed.

As a rule, imposition of additional constraints on a mechanical system reduces its number of degrees of freedom and number of required variables, though entails the necessity to calculate additionally reactions of these constraints (contact forces). So, in the considered case, the functions of the internal moments and shear forces acting in the DS are calculated via simple formulas and can be known. Then, the direct problem of the theory of quasi-statics of curvilinear flexible rods can be stated for determination of internal longitudinal force and torque, while the external forces of contact and friction interaction between the DS and bore-hole wall (the constraint reactions) can be calculated through the statement of an inverse problem. With this approach the behavior of the DS in curvilinear borehole can be described, the zones of possible seizure of the DS can be detected and the measures for the DS jarring and release can be designed. In this case, it is conveniently to use the theory of curvilinear flexible rods to describe the stress-strain state of the DS.

Let geometry of a bore-hole axis line be prescribed and determined by the equation:

$$\boldsymbol{\rho} = \boldsymbol{\rho}(s) \tag{1}$$

in the Cartesian coordinate system $Oxyz$. Here $\boldsymbol{\rho}$ is the radius-vector $\boldsymbol{\rho} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the appropriate unit vectors; s is the parameter measured by the length of the axial line of the bore-hole; $x(s)$, $y(s)$, $z(s)$ are the differentiable functions describing the trajectory of the bore-hole axis with any geometrical imperfections.

To describe mechanics of the drill string inside the borehole, it is suitable to introduce a moving right-handed system of axes (u, v, w), whose orientation is rigidly connected with the cross-section of interest at a generic point of the DS tube axis line. The origin of this system lies at the center of gravity of the cross-sectional area, the u – and v –axes are directed along the principal central axes of inertia of the cross-sectional area, and the w –axis is directed along the tangent to the elastic line.

The Frenet natural trihedron of the elastic line of the rod with unit vectors of the principal normal $\mathbf{n}$, binormal $\mathbf{b}$ and tangent $\boldsymbol{\tau}$ is also introduced, which is determined by the formulas:

$$\boldsymbol{\tau} = \frac{d\boldsymbol{\rho}}{ds}, \qquad \mathbf{n} = R\frac{d\boldsymbol{\tau}}{ds}, \qquad \mathbf{b} = \boldsymbol{\tau} \times \mathbf{n}. \tag{2}$$

Assuming that the tube cross-section dimensions are very small in comparison with its length and curvature radius of its axial line, one can consider the DS geometry to coincide with the bore-hole geometry and the tube material strains to be elastic. Then, the equations of a DS element equilibrium under action of internal forces ($\mathbf{F}$) and moments ($\mathbf{M}$), and external forces ($\mathbf{f}$) and moments ($\mathbf{m}$) can be represented in the form:

$$\frac{d\mathbf{F}}{ds} = -\mathbf{f}, \qquad \frac{d\mathbf{M}}{ds} = -\boldsymbol{\tau} \times \mathbf{M} - \mathbf{m}. \tag{3}$$

Inasmuch as the DS is curvilinear, it is useful to consider equilibrium of its element relative to a reference frame connected with that. Usually for this aim the system of axes u, v, w is used. As this system rotates with its movement along the s axis line, the total derivatives $d\mathbf{F}/ds$, $d\mathbf{M}/ds$ should be expressed in the following form:

$$\frac{d\mathbf{F}}{ds} = \frac{\tilde{d}\mathbf{F}}{ds} + \boldsymbol{\omega}_{\chi} \times \mathbf{F}, \qquad \frac{d\mathbf{M}}{ds} = \frac{\tilde{d}\mathbf{M}}{ds} + \boldsymbol{\omega}_{\chi} \times \mathbf{M}, \tag{4}$$

where $\tilde{d}\mathrm{K}/ds$ is the local derivative; $\boldsymbol{\omega}$ is the vector of the system (u, v, w) rotation velocity when its origin is moving with unit velocity along the bore-hole axis line:

$$\boldsymbol{\omega} = k_R\mathbf{b} + \left(k_T + d\chi/ds\right)\boldsymbol{\tau}. \tag{5}$$

Here k_R and k_T are the absolute curvature and torsion calculated through the formulae:

$$k_R = \left|\frac{d^2\boldsymbol{\rho}}{ds^2}\right|, \qquad k_T = \boldsymbol{\tau} \cdot \left(\mathbf{n} \times \frac{d\mathbf{n}}{ds}\right). \tag{6}$$

Inasmuch as the DS is very long in comparison with its diameter and its reshaping is caused by bending in the bore-hole channel, its elastic elongation can be neglected. Then the forces F_u, F_v, F_w are purely static factors and can be determined from the equilibrium conditions. The components M_u, M_v, M_w of the principal vector $\mathbf{M}$ are calculated with the use of the Hooke law. The initial curvature and torsion of the DS are taken as zero. Then:

$$M_u = Ap, \qquad M_v = Aq, \qquad M_w = Cr, \tag{7}$$

where $A = EI$, $C = GI_w$ are the bending and torsion stiffness factors; E, G – the elastic and shear modules of elasticity; I, I_w – the axial and polar inertia moments of the drill string tube cross-section.

With the above expressions, the curvatures p, q in the (u, v, w) system and elastic torsion r are calculated as:

$$p = k_R \sin\chi, \qquad q = k_R \cos\chi, \qquad r = k_T + \frac{d\chi}{ds}, \tag{8}$$

where χ is the angle between the unit vector $\mathbf{n}$ and axis u.

The geometry parameters determined by Eqs. (1), (2), (5), (6), (8) do not vary with the DS deforming, while the functions $p(s)$, $q(s)$, $r(s)$ evolve at the DS torsion and should be found at every step of its loading.

So, the vectors $\mathbf{F}$, $\mathbf{M}$, $\tilde{d}\mathbf{F}/ds$, $\tilde{d}\mathbf{M}/ds$ and $\boldsymbol{\omega}_\chi$ have the components F_u, F_v, F_w, M_u, M_v, M_w, dF_u/ds, dF_v/ds, dF_w/ds, dM_u/ds, dM_v/ds, dM_w/ds and p, q, r, correspondingly. Then Eqs. (4) can be represented in the scalar form:

$$\begin{aligned} dF_u/ds &= -qF_w + rF_v - f_u, \\ dF_v/ds &= -rF_u + pF_w - f_v, \\ dF_w/ds &= -pF_v + qF_u - f_w \end{aligned} \tag{9}$$

for the force equilibrium and in the same form:

$$\begin{aligned} dp/ds &= \left(Brq - Cqr + F_v - m_u\right)/A, \\ dq/ds &= \left(Cpr - Arp - F_u - m_v\right)/B, \\ dr/ds &= \left(Aqp - Bpq - m_w\right)/C \end{aligned} \tag{10}$$

for the moments equilibrium.

They can be used for modeling the considered quasi-static states. Since the lowering-raising operations and bore-hole drilling are differing each from other by modes of the drill string motion, schemes and values of the external force $\mathbf{f}$ distribution and developing emergency situations, they should be considered individually.

In a general case, the functions $\mathbf{f}(s)$ and $\mathbf{m}(s)$ can be represented through the gravitational force $\mathbf{f}^{gr}(s)$, contact force $\mathbf{f}^{c}(s)$, frictional force $\mathbf{f}^{fr}(s)$, inertia force $\mathbf{f}^{l}(s)$ of the liquid (mud) moving inside the tube, and torsion moment $\mathbf{m}(s)$ as follows:

$$\mathbf{f}(s) = \mathbf{f}^{gr}(s) + \mathbf{f}^{c}(s) + \mathbf{f}^{fr}(s) + \mathbf{f}^{l}(s), \qquad \mathbf{m}(s) = m_w^{fr}(s)\cdot\boldsymbol{\tau}. \tag{11}$$

The gravitation force is known, it has three components:

$$
\begin{aligned}
f_u^{gr} &= -gF(\gamma_t - \gamma_l)(n_z \cos\chi + b_z \sin\chi), \\
f_v^{gr} &= gF(\gamma_t - \gamma_l)(n_z \sin\chi - b_z \cos\chi), \\
f_w^{gr} &= -gF(\gamma_t - \gamma_l)\tau_z,
\end{aligned}
\tag{12}
$$

where F is the area of the cross-section of the drill string, γ_t is the tube material density, and γ_l is the density of the washing liquid.

The contact force $\mathbf{f}^c(s)$ is required one. It is normal to the axial line of the DS and has components f_u^c and f_v^c.

The DS routinely experiences the axial motion with velocity $\dot{w}$ and rotates with angular velocity ω at a time. Then the total friction force as given below:

$$
\left| f^{fr} \right| = \mu \left| f^c \right| = \mu \sqrt{\left(f_u^c \right)^2 + \left(f_v^c \right)^2}
$$

can be decomposed into the axial and circumferential components:

$$
f_w^{fr} = \pm\mu \cdot f^c \frac{\dot{w}}{\sqrt{(\dot{w})^2 + (\omega d_e/2)^2}}, \qquad f_\omega^{fr} = \pm\mu \cdot f^c \frac{\omega d_e}{2\sqrt{(\dot{w})^2 + (\omega d_e/2)^2}}, \tag{13}
$$

which are proportional to the appropriate components of velocities $\dot{w}$ and $\omega d_e/2$. Here d_e is the external diameter of the drill string tube.

The f_w^{fr} force impedes the axial movement of the DS, while the second one acts in the circumferential direction, resulting in the distributed friction moment:

$$
m_w^{fr} = f_\omega^{fr} \cdot \frac{d_e}{2} = \pm\mu \cdot f^c \frac{\omega d_e^2}{4\sqrt{(\dot{w})^2 + (\omega d_e/2)^2}}. \tag{14}
$$

The signs “$\pm$” in Eqs. (13) and (14) should be chosen depending on the directions of movement and rotation of the DS. The sign “–” in the expression for f_w^{fr} corresponds to the procedure of the drill string raising, whereas the sign “+” relates to the processes of lowering and drilling.

The $\mathbf{f}^l$ force is determined as follows:

$$
\mathbf{f}^l = -\pi d_i^2 \gamma_l V^2 k_R \mathbf{n} / 4. \tag{15}
$$

Here d_i is the internal radius of the drill string tube, V is the mud velocity.

As a rule, in solving the practical problems for a curvilinear rod with complicated axial line, it is difficult to choose the S variable, which parameterizes the geometry of the rod. Thus, one is forced to select, instead, some dimensionless parameter ϑ. In this case, the substitution $ds = Dd\vartheta$ should be performed, where D is the metric multiplier calculated by the following formula:

$$D = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\,.$$

Here the dot above a letter denotes differentiation with respect to ϑ.

With these substitutions and introduction of the designations $\chi = h_1$, $d\chi/d\vartheta = dh_1/d\vartheta = h_2$, Eqs. (9), (10) are brought to the form:

$$\frac{dF_u}{Dd\vartheta} = \left(k_T + \frac{h_2}{D}\right)F_v - k_R \cos h_1 \cdot F_w - f_u^{gr} - f_u^{c},$$
$$\frac{dF_v}{Dd\vartheta} = k_R \sin h_1 \cdot F_w - \left(k_T + \frac{h_2}{D}\right)\cdot F_u - f_v^{gr} - f_v^{c},$$
$$\frac{dF_w}{Dd\vartheta} = k_R \cos h_1 \cdot F_u - k_R \sin h_1 \cdot F_v - f_w^{gr} - f_w^{fr}. \quad (16)$$

$$\frac{1}{D}\left[\frac{dk_R}{d\vartheta}\sin \mathrm{h}_1 + k_R \cos \mathrm{h}_1 \cdot h_2\right] = \frac{A-C}{A}\cdot k_R \cos \mathrm{h}_1 \left(k_T + \frac{h_2}{D}\right) + \frac{F_v}{A},$$
$$\frac{1}{D}\left[\frac{dk_R}{d\vartheta}\cos \mathrm{h}_1 - k_R \sin \mathrm{h}_1 \cdot h_2\right] = \frac{C-A}{A}\cdot k_R \sin \mathrm{h}_1 \left(k_T + \frac{h_2}{D}\right) - \frac{F_u}{A},$$
$$\frac{1}{D}\left[\frac{dk_T}{d\vartheta} - \frac{1}{D^2}\frac{dD}{d\vartheta}h_2 + \frac{1}{D}\frac{dh_2}{d\vartheta}\right] = -\frac{m_w^{fr}}{C}. \quad (17)$$

In this system, four functions $F_u(\vartheta)$, $F_v(\vartheta)$, $F_w(\vartheta)$, $h_1(\vartheta)$,which determine the stress-strain state of the DS, are required. The external distributed forces of contact $\left(f_u^c(\vartheta),\ f_v^c(\vartheta)\right)$, friction interaction $\left(f_w^{fr}(\vartheta)\right)$ and friction moment $m_w^{fr}(\vartheta)$ are unknown as well. At the same time, the components f_u^{gr}, f_v^{gr}, f_w^{gr} are considered to be active and given.

The relations brought above make it possible to formulate a set of equations for elastic bending of the DS in a bore-hole with a specified centerline (1). It is finally written as the third order differential equation system:

$$\frac{dh_1}{d\vartheta} = h_2,$$
$$\frac{dh_2}{d\vartheta} = \frac{dD}{Dd\vartheta}h_2 - D\frac{dk_T}{d\vartheta} - \frac{D^2}{C}m_w^{fr},$$
$$\frac{dF_w}{d\vartheta} = D\cdot k_R \cos\mathrm{h}_1 \cdot F_u - D\cdot k_R \sin\mathrm{h}_1 \cdot F_v - D\cdot f_w^{gr} - D\cdot f_w^{fr} \quad (18)$$

together with equalities:

$$F_u = (C-A)k_R k_T \sin h_1 + Ck_R \frac{h_2}{D}\sin h_1 - \frac{A}{D}\frac{dk_R}{d\vartheta}\cos h_1,$$

$$F_v = (C-A)k_R k_T \cos h_1 + Ck_R \frac{h_2}{D}\cos h_1 + \frac{A}{D}\frac{dk_R}{d\vartheta}\sin h_1 \qquad (19)$$

following from Eqs. (17).

Using the equations derived above, one can determine the external and internal force factors initiated in movement of the DS inside a bore-hole with an arbitrary trajectory of axial line. These trajectories can also include any geometrical imperfections, which yet are described by Eq. (1) and are differentiable.

There is a commonly encountered problem in drilling curvilinear boreholes: due to a sudden loss of homogeneity of tectonic structure or violation of the drilling technique, the well trajectories deviates from the designed one and some local geometrical imperfections arise on it. The trajectory perturbations appear usually in the shapes of a three-dimensional spiral, plane harmonic wave or a local smoothed break. This work addresses the break imperfection case. The designed bore-hole centerline is assumed to be represented by a quarter arc of an ellipse with semi axes H and L in the plane xOz:

$$x = L\cos\vartheta, \qquad y = 0, \qquad z = H\sin\vartheta \qquad (3\pi/2 \le \vartheta \le 2\pi). \qquad (20)$$

For this geometry we solve the above-stated problem of determination of internal and external force factors that act on a DS during its motion. Then, the bore-hole centerline geometry is specified to have ~~a~~ distortions in the shape of breaks (Fig. 1).

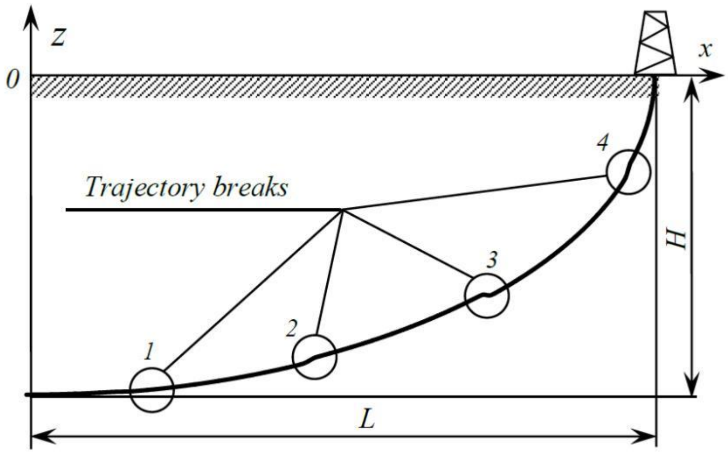

Figure 1 – The bore-hole trajectory with planar imperfections

Рисунок 1 – Траєкторія свердловини з планарними недосконалостями

They can be approximated by superimposing apexes of hyperbolas with different sharp nesses (eccentricities) and angles between their asymptotes:

$$z_1 = -\left\{\frac{\operatorname{tg}\alpha(x-x_1)}{2} + \sqrt{\left[\frac{\operatorname{tg}\alpha(x-x_1)}{2}\right]^2 - \varepsilon}\right\}, \qquad (21)$$

$$z_2 = \left\{\frac{\operatorname{tg}\alpha(x-x_2)}{2} + \sqrt{\left[\frac{\operatorname{tg}\alpha(x-x_2)}{2}\right]^2 - \varepsilon}\right\}. \qquad (22)$$

Here α is the angle between the hyperbola asymptotes, x_i is the x coordinate of the hyperbola center, and ε is the parameter characterizing the hyperbola eccentricity.

If the asymptotes of the left branches of two similar hyperbolas with different signs are oriented along the negative direction of the Ox axis, then their superposition with small shift $\Delta s = x_2 - x_1$ in plane xOz approximately produces a broken rectilinear line with small shift Δs and curvature radii R at the break apexes. If one imposes this line on the initial trajectory as given in Eq. (20), then the ultimate trajectory of the drill string will attain the configuration shown in Fig. 1.

The breaks can be located also out of the xOz plane, then their expressions are as follows:

$$y_{1,2} = \pm\left\{\frac{\mathrm{tg}\,\alpha\left(x - x_{1,2}\right)}{2} + \sqrt{\left[\frac{\mathrm{tg}\,\alpha\left(x - x_{1,2}\right)}{2}\right]^2 - \varepsilon}\right\}. \tag{23}$$

In the general case, the planar and three-dimensional imperfections can be located at several places and have different radii R and angles α. As such, they should be simulated by superposition of different breaks as given in Eqs. (21), (22) or (23) on the design trajectory of Eq. (20).

Integration of system (18), (19) with appropriate initial conditions at $s = 0$ is performed by the Runge-Kutta method.

Results of the research conducted. As noted above, the process of a bore-hole driving includes three major procedures distinguished by technological methods and calculation schemes. The procedure of drilling involves combination of downward axial movement of the drill string with its rotation and is accompanied by action of cutting moment $M_w(0)$ (torque on bit) and reaction $F_w(0)$ (force on bit), as well as inertia force $\mathbf{f}^l(s)$ of the moving liquid. For this procedure all forces (11) must be allowed for and force f_w^{fr} should be taken with the “+” sign.

The procedure of the DS lowering is characterized by absence of the moment $M_w(0)$ and forces $F_w(0)$, $\mathbf{f}^l(s)$, whereas force f_w^{fr} retains its sign “+”.

The procedure of the DS raising differs from the previous one only by the “–” sign before the f_w^{fr} expression.

These procedures were computer simulated under different values of characteristic parameters, namely: $d_e = 0{,}1683m$, $d_i = 0{,}1583m$, $L = 8000m$ and $12000m$, $H = 4000m$, $E = 2{,}1\cdot 10^{11}\,Pa$, $G = 0.8077\cdot 10^{11}\,Pa$, $\gamma_{st} = 7850 kg/m^3$, $\gamma_l = 1500 kg/m^3$, $\nu = 0{,}1$ and 100, $F_w(0) = -5\cdot 10^4\,N$, $M_w(0) = 10^5\,Nm$.

Depending upon formations, typical friction factor used in technological operation simulations ranges from $0{,}22$ in oil base mud to $0{,}35$ in water base mud. In this analysis, the value $\mu = 0{,}2$ was used.

Four geometrical breaks were imposed on the axis line of the bore-hole. Magnitudes of their curvatures were specified in such a way (Fig. 2) as to permit the comparison of their influence on the resistance to the drill string movement.

Firstly, the case $L = 8000m$, $\mu = 0,2$, $\nu = 100$, $V = 0$ was considered. The calculation results are listed in Table 1.

Table 1 – Stress-strain parameters of the drill string bending ($L = 8000m$, $\mu = 0,2$, $\nu = 100$)

Таблиця 1 – Параметри напружено-деформованого стану вигину бурильної колони ($L = 8000м$, $\mu = 0,2$, $\nu = 100$)

Type of operation	Task number	Type of imperfection	$F_w(S)$,*MN*	ΔS , *m*	$M_w(S)$, *Nm*	$\varphi(S)$, *rad*
Drilling	1	None	0,682	0,26	$10{,}042 \cdot 10^4$	384,2
	2	Planar	0,662	0,21	$10{,}046 \cdot 10^4$	387,5
	3	3D	0,588	0,09	$10{,}051 \cdot 10^4$	386,2
Lowering	4	None	0,739	0,77	$0{,}042 \cdot 10^4$	0,90
	5	Planar	0,716	0,74	$0{,}045 \cdot 10^4$	0,94
	6	3D	0,636	0,62	$0{,}051 \cdot 10^4$	0,97
Raising	7	None	1.765	5,74	$0{,}044 \cdot 10^4$	0,82
	8	Planar	2,210	6,95	$0{,}080 \cdot 10^4$	1,22
	9	3D	2,525	7,19	$0{,}108 \cdot 10^4$	1,31

It can be seen that only the break with curvature radius $R \approx 50m$ exerts tangible effect on the functions $F_w(s)$ and $M_w(s)$, whereas three other imperfections do not practically influence on their values.

The resultant bending moment $M_R(s) = \sqrt{M_u^2 + M_v^2}$ depends solely upon the curvature k_R. So, its diagram is similar to the k_R graph (Fig. 2).

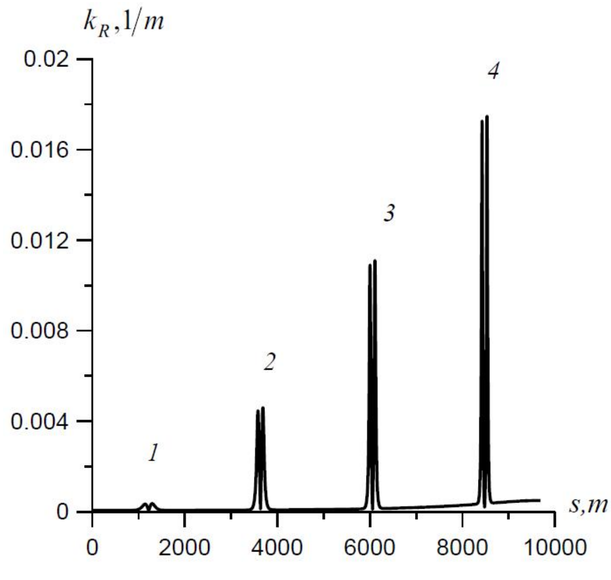

Figure 2 – Diagram of the curvature change

Рисунок 2 – Діаграма зміни кривизни

The friction functions f_w^{fr}, m_w^{fr} also have nearly discontinuous character and rise steeply at the imperfection zones.

The operation of the DS lowering does not qualitatively differ from the drilling, though the values of the $F_w(s)$ and $M_w(s)$ functions assume lesser magnitudes. But the imperfection presence is more conspicuous in the raising operation.

In order to gain more exhaustive insight into the DS stress-strain state, the total elastic elongation of the DS tube:

$$\Delta S = \frac{1}{EF}\int_0^S F_w(s)ds$$

and the angle $\varphi(S)$ of the elastic twist of the DS at the end $s = S$:

$$\varphi(S) = \frac{1}{GI_w}\int_0^S M_w(s)ds$$

are included into the Table data.

Table 2 – Stress-strain parameters of the drill string bending ($L = 8000m$, $\mu = 0,2$, $\nu = 0,1$)

Таблиця 2 – Параметри напружено-деформованого стану вигину бурильної колони ($L = 8000м$, $\mu = 0,2$, $\nu = 0,1$)

Type of operation	Task number	Type of imperfection	$F_w(S)$, *MN*	ΔS, *m*	$M_w(S)$, *Nm*	$\varphi(S)$, *rad*
Drilling	1	None	1,138	2,66	$14,189 \cdot 10^4$	469,9
	2	Planar	1,141	2,69	$15,379 \cdot 10^4$	486,1
	3	3D	1,111	2,64	$16,499 \cdot 10^4$	488,9
Lowering	4	None	1,190	3,13	$4,177 \cdot 10^4$	85,5
	5	Planar	1,190	3,16	$5,524 \cdot 10^4$	100,7
	6	3D	1,159	3,11	$6,720 \cdot 10^4$	105,3
Raising	7	None	1,289	3,62	$4,202 \cdot 10^4$	84,7
	8	Planar	1,325	3,74	$5,855 \cdot 10^4$	103,4
	9	3D	1,325	3,72	$7,256 \cdot 10^4$	108,8

Then, for comparative purposes, the ν parameter was reduced to the value $\nu = 0,1$. It can be seen, that with increase of the DS rotation velocity the internal and external forces exhibit redistribution. Thus, if the DS rotation is slow (Table 1), then the friction forces impede the DS raising but make easier its lowering and the drilling process, while enlargement of the rotation velocity (Table 2) leads to decrease of the axial friction force.

With enlargement of the angular velocity ω, the friction force f_w^{fr}, impeding axial motion of the DS, as well drops and its influence on the system functioning decreases. In this connection, the problem about determination of the axial force $\mathbf{F}(S)$ for a flexible thread inside absolutely smooth cavity with arbitrary prescribed geometry $z = f(x)$ (Fig. 3) is of practical interest.

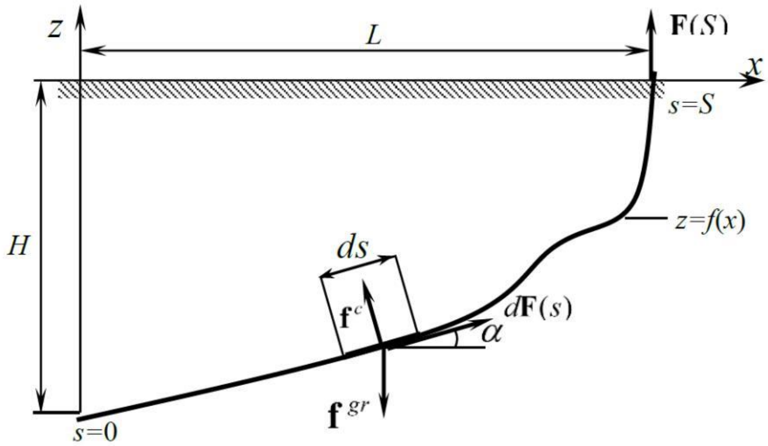

Figure 3 – Equilibrium of a flexible thread in a curvilinear hole

Рисунок 3 – Рівновага гнучкої нитки у криволінійній свердловині

Separate conditionally the thread element of length ds. It is acted upon by the gravity force $\mathbf{f}^{gr}ds$, contact force $\mathbf{f}^{c}ds$ and the force $\mathbf{F}(s)$ increment $d\mathbf{F}$. Project all the forces on the axis tangent to the curve $z = f(x)$:

$$dF - f^{gr} \cdot \sin\alpha \cdot ds = 0 \cdot$$

Then

$$\frac{dF}{ds} = g\gamma \sin\alpha \cdot \qquad (24)$$

Here γ is the mass of the thread unit length, α is the angle of the tangent inclination. It is pertinent to note that Eq. (24) follows also from the third equation of system (9). Use the formula:

$$\sin\alpha = \sqrt{\mathrm{tg}^2\,\alpha / \left(1 + \mathrm{tg}^2\,\alpha\right)} = \sqrt{\left(dz/dx\right)^2 / [1 + \left(dz/dx\right)^2]} \cdot$$

Take into account that:

$$ds = \sqrt{(dx)^2 + (dz)^2} = \sqrt{1 + (dz/dx)^2}\,dx \cdot$$

In this case it issues from Eq.(24):

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$$F(S) = g\gamma \int_0^L \sqrt{(dz/dx)^2 / [1 + (dz/dx)^2]} \cdot \sqrt{1 + (dz/dx)^2} \cdot dx =$$

$$= g\gamma \int_0^L (dz/dx) dx = g\gamma \int_0^H dz = g\gamma H.$$

What this means is that, if to ignore bending stiffness of the DS, then in a smooth curvilinear bore-hole with any geometrical outline the axial force acting on the DS at the suspension point is equal to the gravity force acting on the vertical DS of length H . In our case:

$$F(S) = g(\gamma_{st} - \gamma_l)H = 1,239 MN .$$

Further features of the long bore-hole drilling are essential enlargement of the $M_w(0)$ torque, as well as elastic elongation ΔS and twist angle $\varphi(S)$.

Conclusions. The study involved mathematical modeling of the stress-strain state of a drill string under conditions of a curvilinear wellbore trajectory with potential geometric deviations. The analysis provided a quantitative assessment of key parameters characterizing the mechanical interaction between the pipe and the borehole wall.

1. Impact of trajectory imperfections on friction force and loads: It was established that even small geometric imperfections in the borehole, especially those with a small radius of curvature (e.g., R=100 m), significantly increase the internal and external forces acting on the drill string. This is evidenced by the data in Tables 1 and 2, which compare values of axial forces, elastic elongation, total friction moment and twist angle for various scenarios.
2. Increased load during raising: The most notable increase in loads is observed during the drill string raising (tripping out) operation. For instance, for a 3D imperfection without rotation (Table 1, task 9), the axial force increases to 2,525 *MN* compared to 0,682 *MN* during drilling without imperfections (Table 1, task 1). This indicates that the tripping out operation is the most critical in terms of the risk of sticking or damage to the drill string.
3. Influence of rotation speed: Increasing the drill string's rotation speed leads to a redistribution of internal and external forces. Specifically, with an increase in angular velocity, the axial friction force impeding axial movement decreases, which can facilitate the drilling and lowering processes, but at the same time, it can increase the total friction moment.
4. Elastic elongation and twist: The study shows that the elastic elongation of the drill string and the angle of elastic twist significantly increase in the presence of imperfections and during the tripping out operation. For example, for a 3D imperfection during tripping out (Table 1, task 9), ΔS reaches 7,19 m and $\varphi(S)$ reaches 1,31 rad, which are significantly higher than during drilling without imperfections. These parameters are critical for assessing string integrity and the risk of failure.

The research emphasizes the critical need for developing accurate mathematical models to predict and manage drill string behavior in complex curvilinear boreholes. Existing methods for theoretical simulation of mechanical effects accompanying drilling are insufficiently advanced, making this work particularly relevant.

The developed mathematical model allows for the identification of potential sticking zones for the drill string (so-called "frictional lock-up") through a detailed analysis of contact and frictional interactions. This provides an opportunity to develop preventive measures to avoid emergency situations.

The model can be used to select the most effective drilling, lowering, and raising regimes for the drill string, considering specific geological conditions and borehole geometry. This contributes to increased hydrocarbon extraction efficiency and reduced operating costs.

The authors successfully integrated various factors influencing the stress-strain state of the drill string, including gravity, contact and frictional forces, inertial forces of the fluid, and torque. Such a comprehensive approach ensures more realistic modeling and more accurate predictions.

The study for the first time models the impact of geometric imperfections in the borehole trajectory (breaks approximated by hyperbolas) on drill string dynamics. This is a significant step forward, as such imperfections frequently occur in practice and can lead to serious complications.

The research results have direct practical applications for drilling engineers, enabling them to make informed decisions regarding equipment selection, drilling parameters, and accident prevention strategies. This directly impacts energy security and economic stability.

Conducted research makes a significant contribution to the development of mathematical modeling methods for drilling processes, providing tools to enhance the safety and efficiency of hydrocarbon extraction from deep curvilinear boreholes.

Future research should focus on expanding the mathematical model by incorporating dynamic processes such as drill string vibrations and the effects of variable rock properties and drilling fluid behavior. Special attention should be given to the development of real-time adaptive control of drilling parameters based on simulation results and sensor monitoring data. This approach will enhance the accuracy of predicting emergency situations and support the development of effective drilling management strategies under complex geological conditions.

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МАТЕМАТИЧНА МОДЕЛЬ ТЕХНОЛОГІЧНИХ ОПЕРАЦІЙ БУРІННЯ ГЛИБОКИХ КРИВОЛІНІЙНИХ СВЕРДЛОВИН

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Анотація: У цій статті розглядається процес деформування бурильної колони, під час якого вона вигинається, торкаючись стінок нафто-газової свердловини, і продовжує змінювати свою форму під впливом зростаючої поздовжньої сили, розподіленого контактного навантаження та крутного моменту.

Мета роботи – побудувати нелінійні диференціальні рівняння, що описують контактну взаємодію труби бурильної колони зі стінкою свердловини, запропонувати методику їх чисельного розв'язання та представити результати комп'ютерного моделювання.

Методи дослідження – теоретичні та експериментальні.

Розроблено математичну модель для опису механічних взаємодій бурильної колони зі стовбуром свердловини під час буріння, опускання та піднімання. Модель базується на теорії криволінійних гнучких стержнів і враховує внутрішні та зовнішні силові фактори, зокрема сили контакту та тертя, гравітацію та впливи промивної рідини. Система нелінійних диференціальних рівнянь сформульована для опису стану напружень та деформацій бурильної колони з урахуванням геометричних дефектів траєкторії свердловини. Обчислювальні моделювання аналізують вплив різних параметрів кривини свердловини на поведінку бурильної колони, визначаючи суттєві впливи на згинальні моменти, сили тертя та потенційні ділянки прихоплення бурильної колони.

Практичне застосування отриманих результатів полягає у вдосконаленні методів прогнозування аварійних ситуацій, пов'язаних із заклинюванням бурильної колони, та розробці рекомендацій щодо вибору оптимальних параметрів буріння.

Отримані результати дозволяють оцінити можливість виникнення аварійних ситуацій і оптимізувати режими буріння для підвищення ефективності процесу, мінімізувати зношення обладнання та зменшити ризики виникнення аварій під час освоєння нафтових і газових родовищ.

Ключові слова: криволінійне буріння, свердловина, геометричні недосконалості, внутрішні і зовнішні сили, прямі та обернені задачі.

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