

INVESTIGATION OF THE THERMALLY STRESSED STATE OF SHALLOW SHELLS ON A RIGID BASE WITH A SLIDING CONTACT LAYERS USING ANALYTIC SOLUTIONS OF EQUATIONS OF ELASTICITY THEORY

ДОСЛІДЖЕННЯ ТЕРМОНАПРУЖЕНОГО СТАНУ ПОЛОГИХ ОБОЛОНОК НА ЖОРСТКІЙ ОСНОВІ З КОВЗАЮЧИМ КОНТАКТОМ ШАРІВ ЗА ДОПОМОГОЮ АНАЛІТИЧНОГО РОЗВ'ЯЗАННЯ РІВНЯНЬ ТЕОРІЇ ПРУЖНОСТІ



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Abstract: Modern calculations of layered plates and shells in a three-dimensional formulation are based on a technique where the distribution of the desired functions over the thickness of a structure is sought by the method of discrete orthogonalization. In this article, based on the approaches developed by the authors, the thermally stressed state of layered composite shallow shells with a rigidly fixed lower surface is analyzed. The distribution of the desired functions over the thickness of the structure is found based on the exact analytical solution of the system of differential equations.

An approach to the study of the thermal stress state of shallow composite shells is considered, and an analytical model is constructed for calculating the thermal stress state of shallow shells on a rigid base with a sliding contact of the layers. Currently, this is a very urgent task when calculating the pavement of bridges. A

feature of this approach is the assignment of the desired functions to the outer surfaces of the layers, which allows one to break the layer into sublayers, reducing the approximation error to almost zero. Using the model in question, an analysis of flat layered composite shells on a rigid base with a sliding contact of the layers under the influence of temperature loading was carried out.

To build a spatial model, a load option is selected with temperature loads (according to the sine law) and boundary conditions (Navier), which lead to the distribution of the desired functions in terms of a plate with trigonometric harmonics of the Fourier series. A polynomial approximation of the desired functions by thickness is involved.

Using the model in question, an analysis of flat layered composite shells on a rigid base with a sliding contact of the layers under the influence of temperature loading was carried out. The considered example showed that the proposed model provides sufficient accuracy in the calculations of layered shallow shells when considering each layer within one sublayer.

The proposed approach can be used as a reference method for testing applied approaches in calculating various stress states of layered flat composite shells.

Key words: temperature loads, tangential loads, thermo-stressed state, layered composite shells, semi-analytical finite element method.

Introduction. Calculations of layered plates and shells in a three-dimensional setting, including calculations for temperature effects, are most fully presented in [1]. Modern similar calculations are based on a technique where the distribution of unknown functions over the thickness of the structure is sought by the method of discrete orthogonalization.

In this article, based on the approaches developed by the authors, the thermally stressed state of layered composite sloping shells with a rigidly fixed lower surface is analyzed. The determination of the distribution of the unknown functions over the thickness of the structure occurs on the basis of an exact analytic solution of the system of differential equations.

1. Construction of the solution

In order to construct a spatial model, the variant of the loading by the temperature load (by the law of the sinus) and the boundary conditions (Navier) is chosen, which leads to the distribution of the desired functions in terms of the plate by the trigonometric harmonics of the Fourier series. The polynomial approximation of the desired functions in thickness is involved.

The layered structure in the Cartesian coordinate system is considered. The faces of the layers are evenly spaced at a certain distance and have zero curvature of the torsion $k_{12} = 0$. The variability of the main curvature is neglected $k_{11} = \text{const}$; $k_{22} = \text{const}$. The construction is rather slender, that is, for the coefficients of the first quadratic form should be taken $A_1 \approx l$; $A_2 \approx l$. It is assumed that the radii of curvature are considerably larger than the thickness of the structure and its outer surfaces have the same curvature, it is also possible to write $l + k_{11}z$; $l + k_{22}z \approx l$. The introduced restrictions allow us to identify the curvilinear system of orthogonal coordinates with a planar orthogonal coordinate system. The X axis (lower index 1) and Y (lower index 2) are directed in the plan view, Z axis (lower index 3) for its thickness down; a_{k-1} , a_k – coordinates of external surfaces of k -th layer of the plate along the Z -axis; a comma at the level of the index means a differentiation operation.

The vector of displacement, transverse components of the stress tensor, as well as the temperature distribution in the following form are depicted:

$$\begin{aligned} U_i^{(k)}(x, y, z) &= V_i^{(k)}(x, y) f_i^{(k)}(z); \\ \sigma_{i3}^{(k)}(x, y, z) &= \tau_{i3}^{(k)}(x, y) f_{i+3}^{(k)}(z); \\ T^{(k)}(x, y, z) &= t^{(k)}(x, y) f_7^{(k)}(z), (i = 1, 2, 3). \end{aligned} \quad (1)$$

Using the Cauchy relations, as well as representing the displacements and transverse stresses (1), expressions for deformations:

$$\begin{aligned} e_{ii}^{(k)} &= V_{i,i}^{(k)} f_i^{(k)} + k_{ii} V_3^{(k)} f_3^{(k)}; \\ e_{33}^{(k)} &= V_3^{(k)} f_{3,3}^{(k)}; \\ 2e_{12}^{(k)} &= V_{1,2}^{(k)} f_1^{(k)} + V_{2,1}^{(k)} f_2^{(k)}; \\ 2e_{i3}^{(k)} &= V_i^{(k)} f_{i,3}^{(k)} + V_{3,i}^{(k)} f_3^{(k)}, (i = 1, 2). \end{aligned} \quad (2)$$

Hooke's law and relations for deformations (2) make it possible to determine the longitudinal components of the stress tensor $\sigma_{11}^{(k)}$, $\sigma_{22}^{(k)}$, $\sigma_{12}^{(k)}$ with allowance for the temperature loading (1).

$$\begin{aligned} \sigma_{11}^{(k)} &= B_{11}^{(k)}(V_{1,1}^{(k)} f_1^{(k)} + k_{11} V_3^{(k)} f_3^{(k)}) + B_{12}^{(k)}(V_{2,2}^{(k)} f_2^{(k)} + \\ &+ k_{22} V_3^{(k)} f_3^{(k)}) + B_{13}^{(k)} \tau_{33}^{(k)} f_6^{(k)} - (\alpha_1 B_{11}^{(k)} + \alpha_2 B_{12}^{(k)}) t^{(k)}(x, y) f_7^{(k)}(z); \\ \sigma_{22}^{(k)} &= B_{21}^{(k)}(V_{1,1}^{(k)} f_1^{(k)} + k_{11} V_3^{(k)} f_3^{(k)}) + B_{22}^{(k)}(V_{2,2}^{(k)} f_2^{(k)} + k_{22} V_3^{(k)} f_3^{(k)}) + \\ &+ B_{23}^{(k)} \tau_{33}^{(k)} f_6^{(k)} - (\alpha_1 B_{21}^{(k)} + \alpha_2 B_{22}^{(k)}) t^{(k)}(x, y) f_7^{(k)}(z); \\ \sigma_{12}^{(k)} &= G_{12}^{(k)}(V_{1,2}^{(k)} f_1^{(k)} + V_{2,1}^{(k)} f_2^{(k)}). \end{aligned} \quad (3)$$

Substituting the expressions for deformations (2) and deformations (3) into the variational Reissner principle, following [2], we obtain a resolving system of equations and corresponding boundary conditions. For articulating support (the Navier boundary conditions), if the external loads are given by the following law:

$$\begin{aligned} q_{13}^l(x, y, a_0) &= q_{11} \cos \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; & q_{13}^n(x, y, a_n) &= q_{12} \cos \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; \\ q_{23}^l(x, y, a_0) &= q_{21} \sin \frac{\pi m x}{a} \cos \frac{\pi n y}{b}; & q_{23}^n(x, y, a_n) &= q_{22} \sin \frac{\pi m x}{a} \cos \frac{\pi n y}{b}; \\ q_{33}^l(x, y, a_0) &= q_{31} \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; & q_{33}^n(x, y, a_n) &= q_{32} \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; \end{aligned}$$

$$T_{q3}^l(x, y, a_0) = T_{q3l} \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; \quad T_{q3}^n(x, y, a_0) = T_{q32} \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b}.$$

The distribution of the unknown function in the shell plan is represented thus:

$$V_1^{(k)} = \tau_{13}^{(k)} = \cos \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; \quad V_2^{(k)} = \tau_{23}^{(k)} = \sin \frac{\pi m x}{a} \cos \frac{\pi n y}{b};$$

$$V_3^{(k)} = \tau_{33}^{(k)} = \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b}; \quad t^{(k)} = \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b}.$$

Then the equilibrium equations for the orthotropic layered shell and the known steady-state heat conduction equation, which, in the framework of assumptions about the flatness of the shells, coincides with the well-known equation for the plate

$$\lambda_1^{(k)} T_{,11}^{(k)} + \lambda_2^{(k)} T_{,22}^{(k)} + \lambda_3^{(k)} T_{,33}^{(k)} = 0$$

with considering $Q_3^{(k)}(x, y, z) = -t^{(k)}(x, y) \lambda_3^{(k)} f_{7,3}^{(k)}(z)$ are transformed as follows:

$$f_{1,3}^{(k)} = -f_3^{(k)} \left(\frac{\pi m x}{a} \right) + f_4^{(k)} \left(\frac{I}{G_{13}^{(k)}} \right);$$

$$f_{2,3}^{(k)} = -f_3^{(k)} \left(\frac{\pi n y}{b} \right) + f_5^{(k)} \left(\frac{I}{G_{23}^{(k)}} \right);$$

$$f_{3,3}^{(k)} = f_1^{(k)} B_{13}^{(k)} \left(\frac{\pi m x}{a} \right) + f_2^{(k)} B_{23}^{(k)} \left(\frac{\pi n y}{b} \right) + f_3^{(k)} (k_{11} B_{13}^{(k)} + k_{22} B_{23}^{(k)}) +$$

$$+ f_6^{(k)} B_{33}^{(k)} + f_7^{(k)} (B_{13}^{(k)} \alpha_1^{(k)} + B_{23}^{(k)} \alpha_2^{(k)} + \alpha_3^{(k)});$$

$$f_{4,3}^{(k)} = f_1^{(k)} \left[B_{11}^{(k)} \left(\frac{\pi m x}{a} \right)^2 + G_{12}^{(k)} \left(\frac{\pi n y}{b} \right)^2 \right] +$$

$$+ f_2^{(k)} (B_{12}^{(k)} + G_{12}^{(k)}) \left(\frac{\pi m x}{a} \right) \left(\frac{\pi n y}{b} \right) - f_3^{(k)} \left[k_{11} B_{11}^{(k)} \left(\frac{\pi m x}{a} \right) + k_{22} B_{12}^{(k)} \left(\frac{\pi m x}{a} \right) \right] -$$

$$- f_6^{(k)} B_{13}^{(k)} \left(\frac{\pi m x}{a} \right) + f_7^{(k)} (B_{11}^{(k)} \alpha_1^{(k)} + B_{12}^{(k)} \alpha_2^{(k)}) \left(\frac{\pi m x}{a} \right);$$

$$f_{5,3}^{(k)} = f_1^{(k)} (B_{12}^{(k)} + G_{12}^{(k)}) \left(\frac{\pi m x}{a} \right) \left(\frac{\pi n y}{b} \right) +$$

$$\begin{aligned}
 & + f_2^{(k)} \left[B_{22}^{(k)} \left(\frac{\pi y}{b} \right)^2 + G_{12}^{(k)} \left(\frac{\pi x}{a} \right)^2 \right] - f_3^{(k)} \left[k_{11} B_{21}^{(k)} \left(\frac{\pi y}{b} \right) + k_{22} B_{22}^{(k)} \left(\frac{\pi y}{b} \right) \right] - \\
 & - f_6^{(k)} B_{23}^{(k)} \left(\frac{\pi y}{b} \right) + f_7^{(k)} \left(B_{21}^{(k)} \alpha_1^{(k)} + B_{22}^{(k)} \alpha_2^{(k)} \right) \left(\frac{\pi y}{b} \right); \\
 & f_{6,3}^{(k)} = -f_1^{(k)} \left[k_{11} B_{11}^{(k)} \left(\frac{\pi x}{a} \right) + k_{22} B_{12}^{(k)} \left(\frac{\pi x}{a} \right) \right] - \\
 & - f_2^{(k)} \left[k_{11} B_{12}^{(k)} \left(\frac{\pi y}{b} \right) + k_{22} B_{22}^{(k)} \left(\frac{\pi y}{b} \right) \right] + \\
 & + f_3^{(k)} \left(k_{11}^2 B_{11}^{(k)} + 2k_{11} k_{22} B_{12}^{(k)} + k_{22}^2 B_{22}^{(k)} \right) + \\
 & + f_4^{(k)} \left(\frac{\pi x}{a} \right) + f_5^{(k)} \left(\frac{\pi y}{b} \right) - f_6^{(k)} \left(k_{11} B_{13}^{(k)} + k_{22} B_{23}^{(k)} \right) + \\
 & + f_7^{(k)} \left[\left(B_{11}^{(k)} \alpha_1^{(k)} + B_{12}^{(k)} \alpha_2^{(k)} \right) k_{11} - \left(B_{21}^{(k)} \alpha_1^{(k)} + B_{22}^{(k)} \alpha_2^{(k)} \right) k_{22} \right] + \\
 & + f_7^{(k)} \left(B_{21}^{(k)} \alpha_1^{(k)} + B_{22}^{(k)} \alpha_2^{(k)} \right) \left(\frac{\pi y}{b} \right); \\
 & f_{7,3}^{(k)} = -\frac{1}{\lambda_3^{(k)}} f_8^{(k)}; \\
 & f_{8,3}^{(k)} = -f_7^{(k)} \left[\lambda_1^{(k)} \left(\frac{\pi x}{a} \right)^2 + \lambda_2^{(k)} \left(\frac{\pi y}{b} \right)^2 \right]. \tag{4}
 \end{aligned}$$

The solution of system (4) is found in this form: $f_i^{(k)} = \mu_i^{(k)} e^{\beta^{(k)} z}$, $(i = 1, \dots, 8)$.

The roots of the system of characteristic equations can be real, as well as complex, and

$$\beta_7^{(k)} = -\beta_8^{(k)} = \sqrt{\frac{\lambda_1^{(k)}}{\lambda_3^{(k)}} \left(\frac{\pi x}{a} \right)^2 + \frac{\lambda_2^{(k)}}{\lambda_3^{(k)}} \left(\frac{\pi y}{b} \right)^2}.$$

Now the solution of the system of differential equations (4) can be represented as follows:

$$f_i^{(k)} = \mu_{i1}^{(k)} C_1^{(k)} e^{\beta_1^{(k)} z} + \mu_{i2}^{(k)} C_2^{(k)} e^{\beta_2^{(k)} z} + \mu_{i3}^{(k)} C_3^{(k)} e^{\beta_3^{(k)} z} +$$

$$\begin{aligned}
 & + \mu_{i4}^{(k)} C_4^{(k)} e^{-\beta_1^{(k)} z} + \mu_{i5}^{(k)} C_5^{(k)} e^{-\beta_2^{(k)} z} + \mu_{i6}^{(k)} C_6^{(k)} e^{-\beta_3^{(k)} z} + \\
 & + C_7^{(k)} \mu_{i7}^{(k)} e^{\beta_7^{(k)} z} + C_8^{(k)} \mu_{i8}^{(k)} e^{-\beta_7^{(k)} z}; \\
 & f_7^{(k)} = C_7^{(k)} \mu_{77}^{(k)} e^{\beta_7^{(k)} z} + C_8^{(k)} \mu_{78}^{(k)} e^{-\beta_7^{(k)} z}; \\
 & f_8^{(k)} = C_7^{(k)} \mu_{87}^{(k)} e^{\beta_7^{(k)} z} + C_8^{(k)} \mu_{88}^{(k)} e^{-\beta_7^{(k)} z}, \quad (i = 1, \dots, 6).
 \end{aligned}$$

The constants of integration $C_i^{(k)}$, $i = 1, \dots, 8$ are found from the conjugation conditions of the layers.

2. Results of numerical studies

As an example, the thermo-stressed state of a three-layer hollow shell with the following physical-mechanical characteristics was considered: $E_0 = 1 \text{ MPa}$; $E_1^{(1)} = 172 \cdot 1000 \cdot E_0$;

$$E_2^{(1)} = E_3^{(1)} = 6,9 \cdot 1000 \cdot E_0; \quad G_{12}^{(1)} = G_{13}^{(1)} = 3,45 \cdot 1000 \cdot E_0; \quad G_{23}^{(1)} = 1,38 \cdot 1000 \cdot E_0;$$

$$\nu_{12}^{(1)} = \nu_{13}^{(1)} = \nu_{32}^{(1)} = 0,25 \quad (\nu_{21}/E_2 = \nu_{12}/E_1); \quad \lambda_1^{(1)} = 1,2 \frac{W}{m \cdot \text{deg}};$$

$$\lambda_2^{(1)} = \lambda_3^{(1)} = 0,8 \frac{W}{m \cdot \text{deg}}; \quad \alpha_0 = 1^\circ; \quad \alpha_1^{(1)} = 8,0 \cdot 10^{-6} \cdot \alpha_0;$$

$$\alpha_2^{(1)} = \alpha_3^{(1)} = 163,0 \cdot 10^{-6} \cdot \alpha_0; \quad h^{(1)} = \frac{h}{4} \quad (h = 1 \text{ m}). \text{ The second layer, the thickness}$$

$$h^{(2)} = \frac{h}{2}, \text{ identical to the first, but turned to } 90^\circ, \text{ the third layer is similar to the first, } a/h = 5 \text{ (} a = b$$

). Thermomechanical contact of the layers is perfect. The plate is loaded on the upper surface with a temperature distributed by the law of the sinus $T_{q31} = 50 \cdot T_0$ ($T_0 = 1^\circ$). The bottom surface temperature of zero

$$T_{q32} = 0. \text{ Curvature of the shell } k_{11} = \pm kr1; \quad k_{22} = \pm kr2 \quad (kr1 = \frac{8 \cdot (h/5)}{(a+b)/a};$$

$$kr2 = \frac{8 \cdot (h/5)}{(a+b)/b}). \text{ The calculation was made when considering each layer within a single sublayer. The}$$

value of temperature at the boundaries of the layers with consideration of each layer within a single sublayer: 50; 35.0164; 10.7681; 0.

$$\text{Table 1 shows the dimensionless values of displacements, } \bar{U}_1 = \frac{U_1}{h \cdot \alpha_0 \cdot T_0 \cdot 10^{-4}},$$

$$\bar{U}_3 = \frac{U_3}{h \cdot \alpha_0 \cdot T_0 \cdot 10^{-3}} \text{ as well as stresses } \bar{\sigma}_{11} = \frac{\sigma_{11}}{\alpha_0 \cdot T_0 \cdot E_0} \text{ and } \bar{\sigma}_{22} = \frac{\sigma_{22}}{\alpha_0 \cdot T_0 \cdot E_0} \text{ on the}$$

boundaries of layers of a tight shell with sliding contact of layers and with a rigidly fixed bottom surface.

Considered the variant with negative and positive curvature (Z axis is directed downwards).

Table 1 - Dimensional values of displacements and stresses on the boundaries of layers with sliding contact of layers and with a rigidly fixed bottom surface.

Таблиця 1 - Розмірні значення зміщень і напружень на межі шарів з ковзаючим контактом шарів і з жорстко закріпленою поверхнею дна.

№ layer	$\bar{U}_1$		$\bar{U}_3$		$\bar{\sigma}_{11}$		$\bar{\sigma}_{22}$	
$k_{11} = -kr1; \quad k_{22} = -kr2$								
1	-5.213	-4.195	4.858	-23.06	-5.213	-4.195	4.858	-23.06
	-.5504	-2.317	-31.74	-9.230	-.5504	-2.317	-31.74	-9.230
2	-41.16	-2.317	-20.98	.2993	-41.16	-2.317	-20.98	.2993
	-35.56	-.2969	3.459	-6.160	-35.56	-.2969	3.459	-6.160
3	-.5330	-.2969	-10.14	-11.48	-.5330	-.2969	-10.14	-11.48
	0	0	.2694	.2177	0	0	.2694	.2177
$k_{11} = kr1; \quad k_{22} = kr2$								
1	-8.394	-4.113	-6.854	-24.51	-8.394	-4.113	-6.854	-24.51
	-3.677	-2.235	-23.25	-9.730	-3.677	-2.235	-23.25	-9.730
2	-41.76	-2.235	-21.96	-11.07	-41.76	-2.235	-21.96	-11.07
	-36.27	-2.631	3.482	.0909	-36.27	-2.631	3.482	.0909
3	-.5831	-2.631	-13.00	-11.85	-.5831	-2.631	-13.00	-11.85
	0	0	-.0644	-.0521	0	0	-.0644	-.0521

Conclusions. The approach to the investigation of the thermally stressed state of layered composite shells is developed and a spatial model for calculating the thermally stressed state of flat shells on a rigid basis with sliding contact of layers is constructed. At the present time, this is a very topical task when calculating road pavement bridges. The peculiarity of such an approach is to assign the desired functions to the outer surfaces of the layers, which allows the layers to be broken into substrates, reducing the approximation error to practically zero. Using the considered model, an analysis of flat layered composite shells on a rigid basis with sliding contact of layers under the influence of temperature load was carried out. The considered example showed that the proposed model provides sufficient accuracy in calculations of layered flat shells when considering each layer within a single sublayer. The proposed approach can be used as a reference in the testing of applied approaches.

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Анотація: Сучасні розрахунки шаруватих плит і оболонок в тривимірній постановці засновані на методиці, де розподіл шуканих функцій по товщині конструкції розшукується методом дискретної ортогоналізації. В даній статті на основі підходів, розроблених авторами, проаналізований термонапружений стан шаруватих композитних пологих оболонок з жорстко закріпленою нижньою поверхнею. Знаходження розподілу шуканих функцій по товщині конструкції відбувається на основі точного аналітичного рішення системи диференціальних рівнянь.

Розглянутий підхід до дослідження термонапруженого стану пологих композитних оболонок та побудовано аналітичну модель для розрахунку термонапруженого стану пологих оболонок на жорсткій основі з ковзаючим контактом шарів. На теперішній час це є вельми актуальна задача при розрахунку дорожніх покриттів мостів. Особливістю такого підходу є віднесення шуканих функцій до зовнішніх поверхонь шарів, що дозволяє розбивати шари на підшари, зменшуючи похибку апроксимації практично до нуля. З використанням розглянутої моделі проведений аналіз пологих шаруватих композитних оболонок на жорсткій основі з ковзаючим контактом шарів під впливом температурного навантаження.

Для побудови просторової моделі вибирається варіант навантаження температурним навантаженням (за законом синуса) і граничних умов (Нав'є), який призводить до розподілу шуканих функцій в плані плити за тригонометричними гармоніками ряду Фур'є. Залучається поліноміальна апроксимація шуканих функцій по товщині.

З використанням розглянутої моделі проведений аналіз пологих шаруватих композитних оболонок на жорсткій основі з ковзаючим контактом шарів під впливом температурного навантаження. Розглянутий приклад засвідчив, що запропонована модель забезпечує достатню точність в розрахунках шаруватих пологих оболонок при розгляданні кожного шару в рамках одного підшару.

Запропонований підхід може застосовуватися у якості еталонного методу при тестуванні прикладних підходів в розрахунках різних напружених станів шаруватих пологих композитних оболонок.

Ключові слова: температурні навантаження, дотичні навантаження, термонапружений стан, шаруваті композитні оболонки, напіваналітичний метод кінцевих елементів.

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